Axial crushing of thin-walled structures with origami patterns

Jie Song, Yan Chen*, Guoxing Lu

School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore

ARTICLE INFO

Article history:
Received 26 November 2011
Received in revised form 11 February 2012
Accepted 12 February 2012
Available online 3 March 2012

Keywords:
Energy absorption
Axial crushing
Origami pattern
Thin-walled tubes

ABSTRACT

Thin-walled tubes are a kind of popular design for the energy absorbing devices. However, when they are subjected to axial loading, there exists a large undesirable initial peak force, followed by fluctuation in the force–displacement curve. In this paper, the origami patterns are introduced to thin-walled tubes to minimize the initial peak and the subsequent fluctuations. Tubes of square, hexagonal and octagonal cross-sections with origami patterns are investigated by finite element analysis. Numerical results show that compared with the conventional tube, the patterned tubes exhibit a lower initial peak force and more uniform crushing load. The critical states are obtained under which the crushing mode follows the initial origami pattern. The parametric study shows the relationship between the pre-folding angle and the initial peak force as well as the mean crushing force for the tubes with different cross-sections. A prototype of the patterned tube is constructed and tested, showing much lower initial peak force and a smooth crushing process which agrees with the numerical results.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Thin-walled tubes are widely used as energy absorbers in various industries. For decades many efforts have been made to study their energy absorbing performance under various loading conditions. The axial crushing of thin-walled tubes, in particular those of circular or rectangular cross-section, has been studied extensively. The crushing process of the thin-walled tube can be divided into three stages. First, the crushing force reaches the initial peak to overcome the initial resistance of the tube; second, the force drops and fluctuates around the mean crushing force while the tube deforms; last, the crushing force increases rapidly due to too little space left to compress, marking the end of the crushing process. Usually the crushing process in the second stage is expected to be long and stable for the maximum energy absorption. For many practical applications, the excessively high initial peak force is undesirable because it will lead to a large deceleration with much enhanced probability of damage or human fatality.

Briefly, only the most important work on the theoretical studies of the axial crushing of thin-walled tubes is introduced here. A theoretical model of the circular tube crushed in concertina mode was first proposed by Alexander [1] to calculate the mean crushing force, in which the folds are all external and flattened. The model was later revisied by including the curvature of the folds [2] and the eccentricity factor [3] which took both internal and external folds into account. For rectangular tubes, Wierzbicki and Abramowicz [4] used the super folding element and derived an expression for the mean crushing force. This model underestimates the mean crushing force. Hence, the ultimate tensile stress had to be taken as the flow stress. Later this theory was extended by Abramowicz and Jones [5] with the consideration of effective crushing distance. This model led to a higher mean crushing force, which agrees better with the experiments. The extensional crushing mode of the rectangular tube was investigated theoretically by Abramowicz and Jones [6] to study the mixed crushing modes of a square tube under static and dynamic loading theoretically and experimentally, consisting of both symmetric and extensional components. For tubes with other cross-sectional shapes, Abramowicz and Wierzbicki [7] proposed an improved crushing mode of super folding element to predict the mean crushing force of multi-corner prismatic tubes, in particular the hexagonal tube and the rhomboidal tube. The mean crushing force of octagonal tube was theoretically studied and experimentally verified by Mamalis et al. [8].

Numerical studies on the axial crushing behavior have been performed for tubes with circular and polygonal shapes. Tasdemirci [9] investigated the effect of end constraining condition on the crushing behavior of the circular tube. Langseth et al. [10] studied the dynamic response of the square tube under different impact mass and velocity. Tubes with octagonal cross-section under crushing was simulated and experimentally validated by Mamalis et al. [11]. A comparison in the crushing behavior among square, hexagonal and octagonal tubes subject to dynamic impact was numerically conducted by Rossi et al. [12].

By introducing patterns to the thin-walled tube, the crushing mode can be altered so that better energy absorbing performance
may be achieved. Singace and E1-Sobky [13] studied the effect of corrugations on the crushing behavior of circular tubes. Their results showed that more uniform load–displacement curve and lower initial peak were obtained while the total energy absorption was not improved. Zhang et al. [14] numerically studied the crushing of square tubes with two patterns constructed using the pyramid elements, whose absorbed energy increased by 15–33% and 54–93%, respectively. Zhang and Huh [15] investigated the axial crushing characteristics of longitudinally grooved square tubes numerically. Their results showed that the energy absorption increased by up to 92% and the peak force was reduced by up to 22%, compared with the conventional square tubes.

Introducing origami patterns to thin-walled tubes offers two advantages. First, the initial peak force can be reduced and may be controlled. Second, such tubes exhibit much less fluctuation in the force during crushing. In this paper, we study the axial crushing of thin-walled tubes with origami pattern that have square, hexagonal and octagonal cross-sections. In Section 2, we introduce the geometry of the pattern in a cylindrical. In Section 3, commercial FE code Abaqus/Explicit is adopted to simulate the axial crushing of patterned and conventional tubes. The effect of the pattern on the crushing behavior is analyzed based on numerical results. The axial crushing tests on a prototype are conducted and the results are briefly discussed in Section 4. Finally, some conclusions and discussion are drawn in Section 5.

2. Origami patterned tubes

Origami technique is employed here to generate patterns on thin-walled structures. For example, Yoshimura Pattern (Diamond Pattern) has been applied commercially in the beverage cans in order to save the material as well as make the folding of empty cans easy [16]. Guest and Pellegrino [17–19] investigated triangulated foldable cylinder for the deployable mast structures. Nojima [20] has proposed a large number of patterns for the resultant structures.

Introducing origami patterns to thin-walled tubes offers two advantages. First, the initial peak force can be reduced and may be controlled. Second, such tubes exhibit much less fluctuation in the force during crushing. In this paper, we study the axial crushing of thin-walled tubes with origami pattern that have square, hexagonal and octagonal cross-sections. In Section 2, we introduce the geometry of the pattern in a cylindrical. In Section 3, commercial FE code Abaqus/Explicit is adopted to simulate the axial crushing of patterned and conventional tubes. The effect of the pattern on the crushing behavior is analyzed based on numerical results. The axial crushing tests on a prototype are conducted and the results are briefly discussed in Section 4. Finally, some conclusions and discussion are drawn in Section 5.

2. Origami patterned tubes

Introducing origami patterns to thin-walled tubes offers two advantages. First, the initial peak force can be reduced and may be controlled. Second, such tubes exhibit much less fluctuation in the force during crushing. In this paper, we study the axial crushing of thin-walled tubes with origami pattern that have square, hexagonal and octagonal cross-sections. In Section 2, we introduce the geometry of the pattern in a cylindrical. In Section 3, commercial FE code Abaqus/Explicit is adopted to simulate the axial crushing of patterned and conventional tubes. The effect of the pattern on the crushing behavior is analyzed based on numerical results. The axial crushing tests on a prototype are conducted and the results are briefly discussed in Section 4. Finally, some conclusions and discussion are drawn in Section 5.

Fig. 1. (a) The mobile assembly of spherical 4R linkages; (b) the corresponding crease pattern for rigid origami; (c) the closed tube.


Fig. 2. (a) The crease pattern with equilateral trapezoid; (b) the closed tube in front view; (c) the closed tube in top view.

<table>
<thead>
<tr>
<th>Sample</th>
<th>M</th>
<th>N</th>
<th>L_N</th>
<th>θ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQU-0</td>
<td>4</td>
<td>–</td>
<td>–</td>
<td>180</td>
</tr>
<tr>
<td>SQU-4-170</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>170</td>
</tr>
<tr>
<td>SQU-4-160</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>160</td>
</tr>
<tr>
<td>SQU-4-140</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>140</td>
</tr>
<tr>
<td>SQU-4-120</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>120</td>
</tr>
<tr>
<td>SQU-1-179</td>
<td>4</td>
<td>1</td>
<td>120</td>
<td>179</td>
</tr>
<tr>
<td>SQU-2-179</td>
<td>4</td>
<td>2</td>
<td>60</td>
<td>179</td>
</tr>
<tr>
<td>SQU-3-177</td>
<td>4</td>
<td>3</td>
<td>40</td>
<td>177</td>
</tr>
<tr>
<td>SQU-4-167</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>167</td>
</tr>
<tr>
<td>SQU-5-158</td>
<td>4</td>
<td>5</td>
<td>24</td>
<td>158</td>
</tr>
<tr>
<td>HEX-0</td>
<td>6</td>
<td>–</td>
<td>–</td>
<td>180</td>
</tr>
<tr>
<td>HEX-4-174</td>
<td>6</td>
<td>4</td>
<td>30</td>
<td>174</td>
</tr>
<tr>
<td>HEX-4-170</td>
<td>6</td>
<td>4</td>
<td>30</td>
<td>170</td>
</tr>
<tr>
<td>HEX-4-160</td>
<td>6</td>
<td>4</td>
<td>30</td>
<td>160</td>
</tr>
<tr>
<td>OCT-0</td>
<td>8</td>
<td>–</td>
<td>–</td>
<td>180</td>
</tr>
<tr>
<td>OCT-4-177</td>
<td>8</td>
<td>4</td>
<td>30</td>
<td>177</td>
</tr>
<tr>
<td>OCT-4-170</td>
<td>8</td>
<td>4</td>
<td>30</td>
<td>170</td>
</tr>
<tr>
<td>OCT-4-160</td>
<td>8</td>
<td>4</td>
<td>30</td>
<td>160</td>
</tr>
</tbody>
</table>

3. Numerical simulation

3.1. FE modeling

The FE code Abaqus/Explicit 6.9.2 was adopted to simulate the crushing process. Three-node shell element S3R was used to mesh the tubes, with seven integration points through the thickness. The approximate global element size was 1 mm.

In the simulation, the tube was rested on the fixed base plate. The top plate, initially just contacting the top edge of the tube, moved downwards to crush the tube. Both the plates were modeled as rigid body. Quasi-static procedure was used with crushing distance equal to 73% of the tube length. To ensure quasi-static process, it is desirable that the crushing time is more than 10 times the period of the lowest mode of the structure [22]. In the present study, the time of each sample was determined based on modal analysis and was more than 100 times the sample’s longest period.

The tube was compressed freely and axially in a dry condition. Penalty friction formulation was adopted in the simulation with the Coulomb friction coefficient equal to 0.3. Hard contact model was used to model the contact pressure between surfaces. Small imperfection was introduced to the conventional tubes to trigger its compact crushing mode. For patterned tubes, no imperfection was introduced.

The shell material was mild steel in its annealed state, with the mechanical properties as follows: density \( \rho = 7332.3 \text{ kg/m}^3 \), Young’s modulus \( E = 190.5 \text{ Gpa} \), Poisson’s ratio \( \nu = 0.3 \), yield stress \( \sigma_y = 287.9 \text{ MPa} \) and ultimate stress \( \sigma_u = 506.9 \text{ MPa} \). The strain hardening effect was approximated by using power law hardening model with the strain hardening exponent, \( n = 0.22 \). No strain rate effect was considered.

When the pre-folding angle, \( \theta \), is small enough, i.e., there is large amount of pre-folding, the patterned tube is crushed in the mode prescribed by the pattern. For large value of \( \theta \), especially close to 180°, the pattern may be corrupted during crushing process and the tube deforms in another mode. Therefore, there exists a critical value of \( \theta \) defining the transition condition for the tube to collapse following the initially introduced pattern, when the pre-folding angle is not larger than it. Since continuous value cannot be used for simulation, the critical value was determined with 1° interval. In Table 1, the sample with the value of \( \theta \) shown in bold has \( \theta \) at its own critical value.

3.2. Results

3.2.1. Conventional square tube

Sample SQU-0 was crushed in symmetric mode as shown in Fig. 3(a). The load–displacement (\( P – d \)) curve is shown in Fig. 4. The initial peak of the crushing force \( (P_i) \) and the mean crushing force of simulation \( (P_m^i) \) are shown in Fig. 5. \( P_m^i \) was calculated by

\[
P_m^i = \frac{E_i}{0.73L}
\]

in which \( E_i \) is the total strain energy of the tube, which is given by Abaqus as an output.

3.3. Crushing of patterned tubes

Samples SQU-4-170 to SQU-4-120 are discussed first. Fig. 3(b) and (c) show the profiles of SQU-4-170 and SQU-4-160 at different stages of crushing process, in which SQU-4-170 did not follow the pattern and SQU-4-160 did. The other two samples had similar crushing behavior of SQU-4-160. Fig. 4 shows the load–displacement curves of these samples, and Fig. 5 shows the initial peak and mean crushing force of simulation for these samples.

It is shown in Figs. 4 and 5 that the initial peaks of patterned tubes are much lower than that of the conventional patternless tube, by ranging from 35% to 76%. The initial peak decreases as \( \theta \) decreases. This is due to the compressive membrane deformation of the thin-walled shell. For large \( \theta \), the crushing force is nearly parallel to the shell surface and the shell mainly undergoes membrane compression.
before buckling or collapse, i.e., the membrane force contributing largely to the initial crushing force. As $\theta$ decreases, the membrane compression reduces while the bending deformation increases, making the shell less efficient in resisting crushing. Also from Fig. 4, we can tell that as $\theta$ decreases, the crushing process becomes more uniform. Moreover, the displacement corresponding to the initial peak gradually increases and the initial peak becomes close to subsequent peaks in the crushing process.

Fig. 3. Profiles of square tubes at different stages of crushing process: (a) SQU-0; (b) SQU-4-170; (c) SQU-4-160; (d) SQU-1-179; (e) SQU-2-179; (f) SQU-3-177; (g) SQU-4-167; (h) SQU-5-158.

Fig. 4. Load–displacement curves of samples SQU-0, SQU-4-170 to SQU-4-120.

Fig. 5. Initial peak and mean crushing force from simulation of samples SQU-0 and SQU-4-170 to SQU-4-120.
Fig. 5 shows that the mean crushing forces of the patterned tubes are lower than that of the conventional tube because the patterned tube has already been “partially folded” when the patterned was initially introduced.

The critical pre-folding angle for the square tubes with $N = 1–5$ has been found as samples SQU-1-179 to SQU-5-158. Fig. 6 shows the load–displacement curves of these samples, along with the conventional tube. Fig. 3(d)–(h) show the profiles of samples SQU-1-179 to SQU-5-158 at different stages of crushing process. For SQU-1-179 and SQU-2-179, the layer height is larger than the plastic wave length of the conventional square tube. In the early stage of crushing process, the deformation followed fold lines of the pattern; however in the late stage, deformation similar to the mode of super folding element occurred at the top end where no fold line was present. Samples SQU-3-177, SQU-4-167 and SQU-5-158, with $\theta$’s at critical values, followed the pattern precisely.

3.3.1. Crushing of tubes of hexagonal and octagonal cross-section

Both hexagonal and octagonal conventional tubes were crushed in extensional mode. All the patterned tubes followed the pattern during the crushing process, with HEX-4-174 and OCT-4-177 at critical states. Figs. 7 and 8 show the profiles of HEX-0, OCT-0, HEX-4-174 and OCT-4-177 at different stages of crushing process. The crushing behaviors of other patterned tubes were similar to that of HEX-4-174 or OCT-4-177, depending on the cross-sectional shape. For all the samples, the load–displacement curves are shown in Fig. 9, and the initial peak and mean crushing force from simulation are shown in Fig. 10.

Similar to the square patterned tubes, the initial peaks of hexagonal and octagonal patterned tubes are lower than those of HEX-4-174 or OCT-4-177. For all the samples, the load–displacement curves are shown in Fig. 10.
the conventional tubes, decreasing by 25–46% for the hexagonal tubes and 3–35% for the octagonal tubes, and the initial peak reduces with the decrease of $\theta$. The patterned tubes have more uniform crushing process. The mean crushing force of the patterned tube decreases as $\theta$ decreases. For all the hexagonal patterned tubes, the mean crushing force is lower than that of the conventional tube. For octagonal patterned tubes, mean crushing forces of samples OCT-4-177 and OCT-4-170 are 28% and 10% higher than the conventional tube, respectively. This suggests that the pattern works better for tubes with a large number of sides. The reason is that for those tubes, the extensional mode is the natural mode and the crushing mode described by the pattern, which requires more energy to trigger, does not occur naturally. By introducing the pattern with $\theta$ large enough but lower than the critical value, we force those tubes to be crushed in the pattern’s mode and higher mean crushing force can be achieved.

### 4. Testing

A prototype of patterned square tube was fabricated and tested under axial loading to demonstrate the feasibility of the ideas presented earlier. The tube was made of mild steel and it had $L=120$ mm, $D=60$ mm, $t=1.5$ mm, $\theta=150^\circ$, $M=4$ and $N=4$. The tube was fabricated by stamping the pattern on two pieces of sheet metal and then welding them together. The completed tube is shown in Fig. 11 and the tube was annealed to remove any residual stress before the axial crushing test.

Using an Instron Universal Testing Machine (type 5500), the tube was crushed at a quasi-static loading speed of 10 mm/min. The test was stopped when the crushing distance reached 87.4 mm. Fig. 12 shows the load–displacement curve and

![Fig. 11. Completed patterned square tube after welding (side and top views).](image1)

![Fig. 12. (a) Load–displacement curve of the square patterned tube in axial test; (b) deformation at different displacements.](image2)
successive deformed shape of the tube at different displacements. It is shown that the tube started to collapse at the top of the tube, followed by alternative inward and outward folds according to its pattern. The initial peak was 11.85 kN and the mean crushing force was 10.85 kN. A smooth crushing force history was observed in the test, as numerical simulation has predicted. A sectioned view of the crushed tube is shown in Fig. 13, where neat folding pattern can be observed. The welded joints were strong enough and did not crack during the crushing.

5. Conclusions

The idea of introducing origami patterns to thin-walled tubes has been proposed and the axial crushing behavior of such tubes has been studied. FE code Abaqus/Explicit has been employed to simulate the crushing process of tubes with square, hexagonal and octagonal sections. The results show that patterned tubes have lower initial peaks and more uniform crushing processes than conventional tubes, making them more suitable as energy absorbers, especially for easy crushing applications. With the optimized design parameters, the mean crushing force of the patterned tube can be higher than that of the conventional one.

Especially, for square tubes, the geometry of the patterned tubes is similar to the pre-folded super folding element in Wierzbicki and Abramowicz’s model. Therefore, when the square patterned tube with $N=2$ layers of patterns, the tube is crashed with the same mode as the conventional patternless tube, which is the reason that the critical pre-folding angle for the sample $N=2$ is very close to 180°.

A prototype of the patterned square tube was fabricated and tested. The results showed that the crushing of the tube followed the pattern and a smooth crushing force history was observed, which agreed well with the numerical results.

Acknowledgment

We thank undergraduate student Mr Goh Yu Qian Ivan for his help in fabricating the prototype of the patterned tube and conducting the axial crushing test.

References