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A family of mixed double-Goldberg 6*R* linkages

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A complete family of double-Goldberg 6*R* linkages is reported in this article by combining a subtractive Goldberg 5*R* linkage and a Goldberg 5*R* linkage through the common link-pair or common Bennett-linkage method. A number of distinct types of overconstrained linkages are built, namely the mixed double-Goldberg 6*R* linkages. They all have one degree of freedom and their closure equations are derived in detail. One of them degenerates into a Goldberg 5*R* linkage. From the construction process and geometry conditions, the corresponding relationship between the newly found 6*R* linkages and the double-Goldberg 6*R* linkages, constructed from two Goldberg 5*R* linkages or two subtractive Goldberg 5*R* linkages, has been established.

Keywords: Goldberg 5*R* linkage; double-Goldberg 6*R* linkage; overconstrained linkage; common link-pair method; common Bennett-linkage method

1. Introduction

Various research has been devoted to the invention of single degree-of-freedom overconstrained linkages by combining two or more existing overconstrained linkages. Among them, the Bennett linkage has been a popular construction element since it was proposed in 1903 (Bennett 1903, 1914). Myard (1931) was the first to form 5*R* and 6*R* overconstrained linkages with two Bennett linkages. Later, Goldberg (1943) built a family of 5*R* and 6*R* linkages with two or three Bennett linkages.

For the 5*R* linkages, Baker (1979) re-examined both Myard linkages and the Goldberg 5*R* linkage. He pointed out that the former can be considered as a special case of the latter. A generalized Goldberg 5*R* linkage, initially introduced by Goldberg, was derived by Wohlhart (1991*a*) in detail. Recently, Lee (2002) gave an investigation into the kinematics of the generalized Goldberg 5*R* linkage. Song & Chen (2011) proposed a subtractive Goldberg 5*R* linkage and investigated its kinematic properties.

For the 6*R* linkages, several linkages were found by different researchers using a combination construction method. Waldron (1968) merged two Bennett linkages on a common joint and constrained the relative positioning of the links from these two Bennett linkages to build one of his hybrid 6*R* linkages. Yu & Baker (1981)

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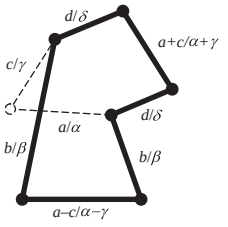
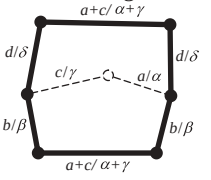
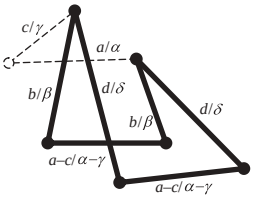
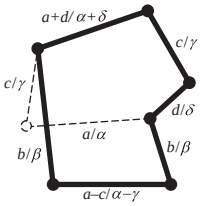
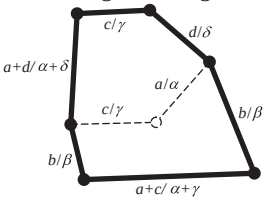
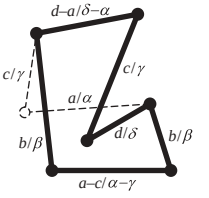
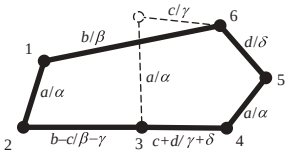
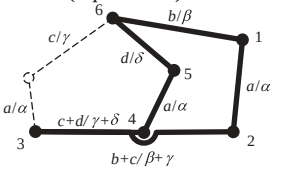
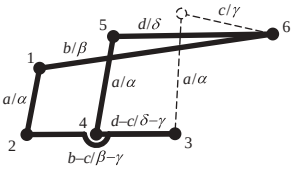
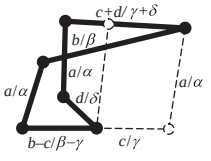
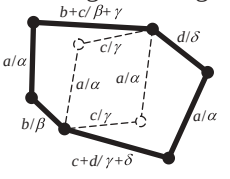
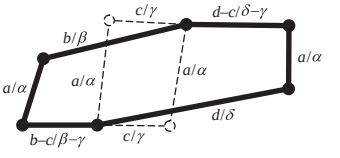
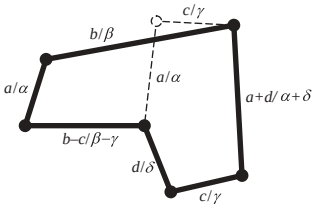
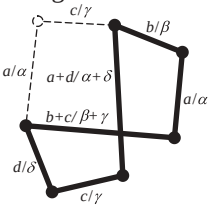
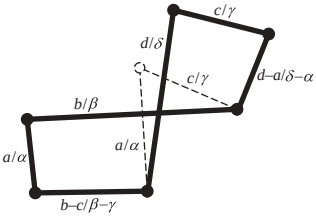
Table 3. The complete families of double-Goldberg 6*R* linkages.

type	the mixed double-Goldberg linkage from linkages S and G	the double-Goldberg linkage from two G linkages	the subtractive double-Goldberg linkage from two S linkages
I		Wohlhart's double-Goldberg 6*R* linkage 	
II		Baker's first variant of Goldberg 6*R* linkage 	
III		Goldberg 5*R* linkage (equivalent) 	
IV		Baker's second variant of Goldberg 6*R* linkage 	
V		one of Goldberg's 6*R* linkage variations 	

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